GV103: Introduction to International Relations

Dr. Philip Arena

Game Theory

Introduction

- Three goals for this lecture
  1. Introduce some terms and concepts
  2. Show you how to identify equilibria in simple models
  3. Analyze an example of a more sophisticated model

Terminology I

Utility

A subjective measure of how much value an actor attaches to an outcome or expects to receive from pursuing a strategy. Sometimes referred to as payoff. Formally, i's utility for outcome z is denoted $u_i(z)$, or $u_i(s)$ for strategy s.

Strategy

A detailed plan that specifies what actions will be taken at all junctures, regardless of whether they are actually reached.

Terminology II

Normal Form Game

A game-theoretic model in which two or more players must choose their strategies simultaneously. Typically represented by a matrix.

Extensive Form Game

A game-theoretic model in which two or more players make decisions sequentially. Typically represented by a decision tree.

Terminology III

Equilibrium

A set of strategies (and, where relevant, beliefs) that leaves no player with an incentive to unilaterally deviate, and thus identifies outcomes which are stable.

Backwards Induction

A technique for identifying equilibria in extensive form games, whereby decisions are analyzed in reverse order and players are assumed to be forward-looking.

Terminology IV

Incomplete Information

A property of game-theoretic models in which one or more players is uncertain about one or more payoffs for another player.

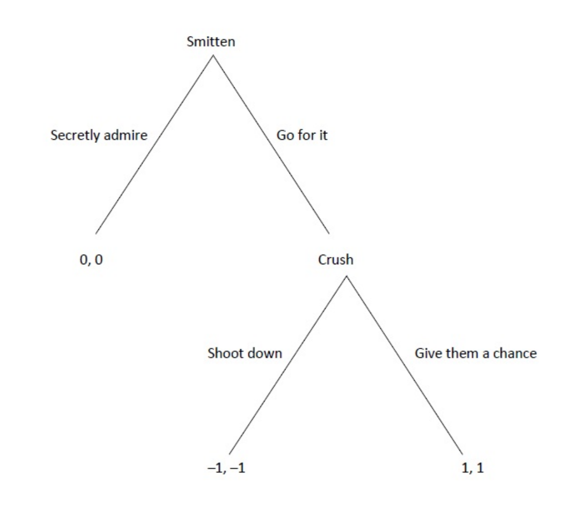
Imperfect Information

A property of game-theoretic models in which one or more players is uncertain about what actions have taken place previously.

Stag Hunt

	Stag	Hare
Stag	2, 2	0, 1
Hare	1, 0	1, 1

Teen Angst


Smitten
Secretly admire
Go for it
0, 0
Crush
Shoot down
Give them a chance
−1, −1
1, 1

Decision Under Uncertainty

- Suppose an actor faces a choice between actions A and B
  - A always yields the same outcome, worth x
  - B yields zx with probability $\frac{1}{z}$, 0 otherwise
  - A risk-acceptant actor strictly prefers B
  - A risk-averse actor strictly prefers A
  - A risk-neutral actor values both equally
  - We'll assume risk-neutrality
- Now must calculate expected utilities, denoted $E(u(s))$
  - For risk-neutral actors, $E(u(s)) = \sum_{i=1}^{N} p_i o_i$
  - If only two possible outcomes, $E(u(s)) = po_1 + (1-p)o_2$

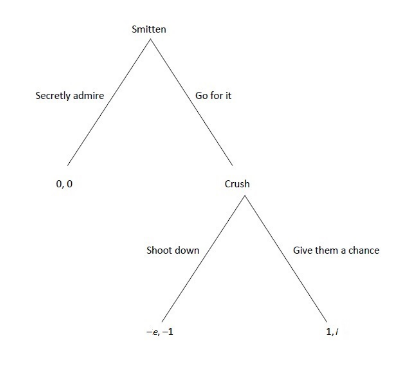
Numerical Examples

- Suppose again there is a choice between A and B
- Where A always yields 0.60
- B yields b_1 w/ probability p, b_2 w/ probability $1 - p$
- How do b_i and p influence the choice of A versus B?

b_1, b_2	p	$E(u(B))$	Choose A?
0.70, 0.10	0.1	0.16	Yes
0.70, 0.10	0.5	0.40	Yes
0.70, 0.10	0.9	0.64	No
0.70, 0.55	0.1	0.57	Yes
0.70, 0.55	0.5	0.63	No
0.70, 0.55	0.9	0.69	No

Teen Angst Revisited

- Let $e > 0$ be the sensitivity of Smitten's ego
- Let $i \in [-\infty, \infty]$ be the level of Crush's interest
- Smitten only knows $pr(i = \bar{i}) = \phi$ and $pr(i = \underline{i}) = 1 - \phi$
- Where $\underline{i} < -1$ and $\bar{i} > 0$



Analysis

- Smitten now compares $u_S(admire) \geq E(u_S(go))$
- Equivalent to $0 \geq \phi(1) + (1 - \phi)(-e)$
- $\Rightarrow 0 \geq \phi - e + \phi e$
- $\Rightarrow e \geq \phi(1 + e)$
- $\Rightarrow \frac{e}{1+e} \geq \phi$
- $\Rightarrow \phi \leq \frac{e}{1+e}$
- Or $\phi \leq \hat{\phi}$ where $\hat{\phi} \equiv \frac{e}{1+e}$