GV103: Introduction to International Relations

Dr. Philip Arena

Mathematical Preliminaries

Introduction

- Four goals for this lecture
  1. Introduce some basic terms and concepts
  2. Discuss measurement of political phenomena
  3. Explain calculation & importance of expected values
  4. Review rules of arithmetic and algebra

Terminology I

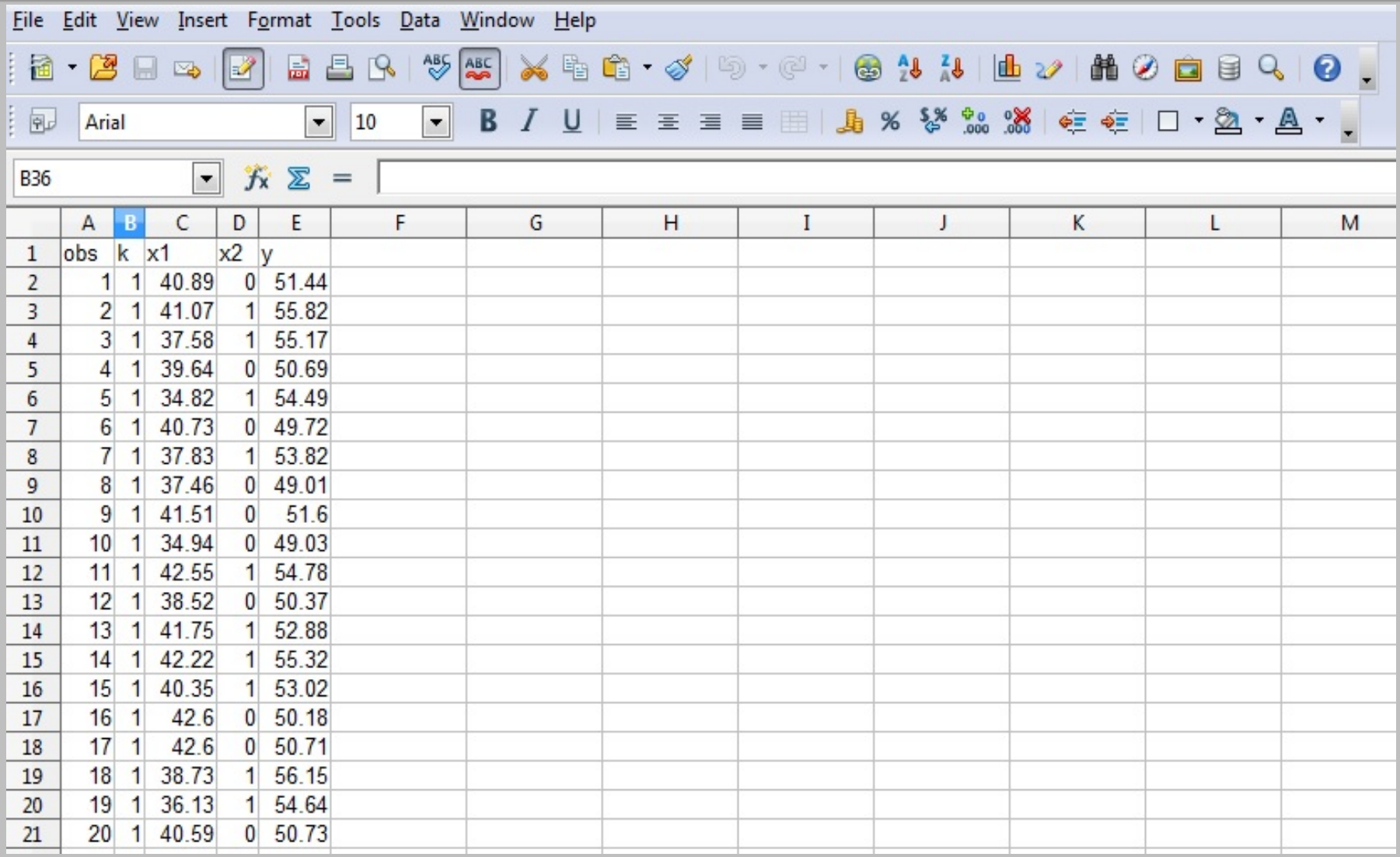
Variable

An alphabetic character, Greek letter, or word that represents numeric values which differ across observations.

Constant

An unchanging numeric value, sometimes represented with an alphabetic character when the value is arbitrary or unspecified.

Example



	A	B	C	D	E	F	G	H	I	J	K	L	M
1	obs	k	x1	x2	y								
2	1	1	40.89	0	51.44								
3	2	1	41.07	1	55.82								
4	3	1	37.58	1	55.17								
5	4	1	39.64	0	50.69								
6	5	1	34.82	1	54.49								
7	6	1	40.73	0	49.72								
8	7	1	37.83	1	53.82								
9	8	1	37.46	0	49.01								
10	9	1	41.51	0	51.6								
11	10	1	34.94	0	49.03								
12	11	1	42.55	1	54.78								
13	12	1	38.52	0	50.37								
14	13	1	41.75	1	52.88								
15	14	1	42.22	1	55.32								
16	15	1	40.35	1	53.02								
17	16	1	42.6	0	50.18								
18	17	1	42.6	0	50.71								
19	18	1	38.73	1	56.15								
20	19	1	36.13	1	54.64								
21	20	1	40.59	0	50.73								

Terminology II

Probability

A measure of how likely something is to occur.Typically written as $pr(x)$ and expressed in decimal form.

Conditional Probability

A measure of how likely something is to occur given a set of conditions. Typically written as $pr(x|c)$.

Levels of Measurement

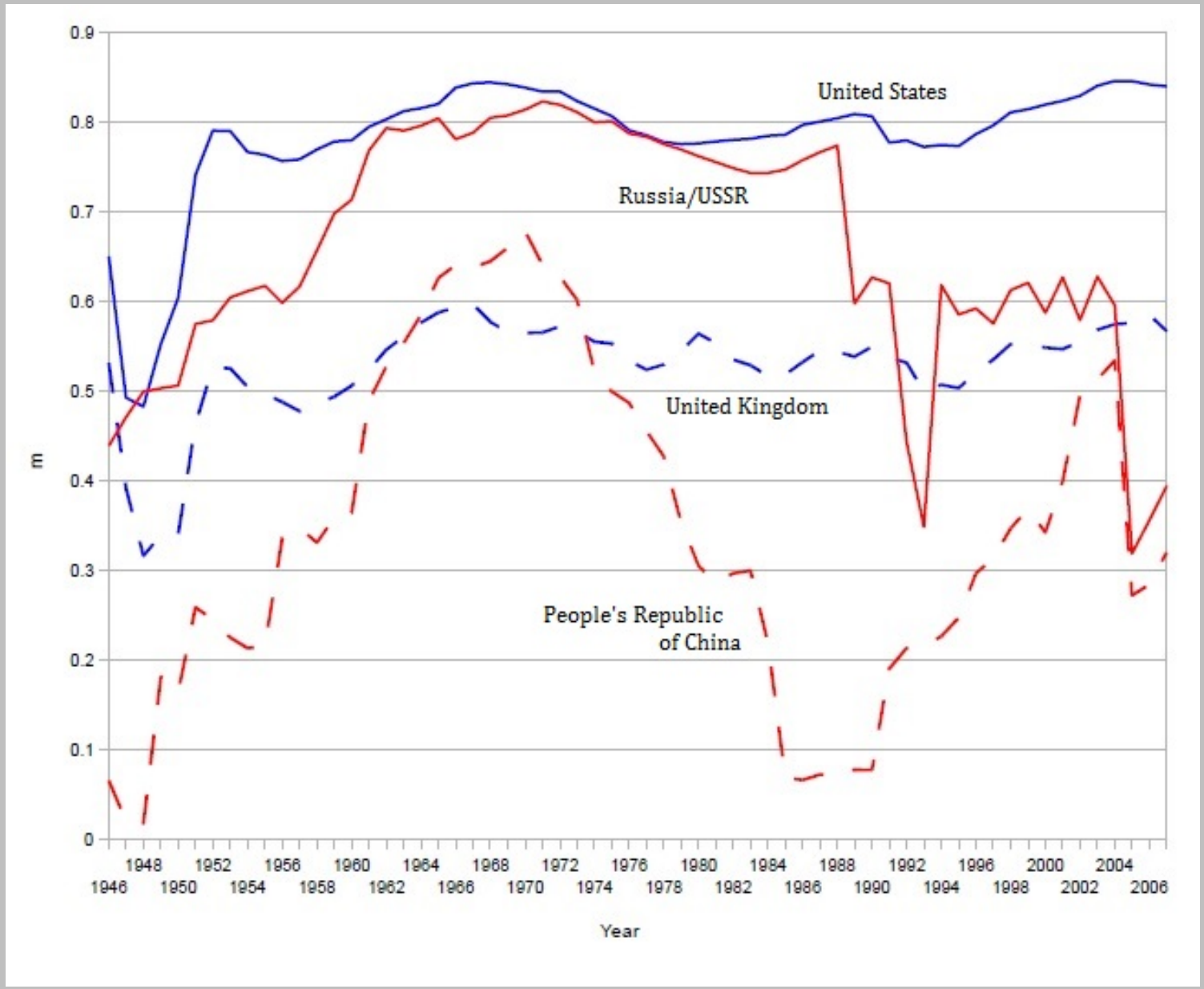
- Variables can be measured at three different levels
  - Nominal
  - Ordinal
  - Interval/Ratio

- Some variables incorporate multiple individual components
  - Indexes
  - Predicted values/probabilities

Examples

- Power
  - Conceptually, ability to alter others' behavior
  - Cannot be measured directly
  - We can measure material factors that likely grant power
  - *m* scores, CINC, GDP
- Democracy
  - Conceptually, governance by the people
  - No consensus on relative importance of process, outcomes
  - Polity, V-Dem, binary measures

A Look at the *m* Scores



Expected Value

- Let x be a random variable
- Each of N outcomes occurs w/ probability p_i and has value z_i
- The expected value of x is denoted $E(x)$
- And is equal to $\sum_{i=1}^{N} p_i z_i$
- Which can also be written as $p_1 \times z_1 + p_2 \times z_2 + \ldots + p_N \times z_N$

Example: Expected Payout of a Bet

- You and a friend place a wager on the outcome of an election
- Friend agrees to pay £20 if long shot wins
- You will owe £10 if the candidate/party that is ahead wins
- Long shot estimated to have 35% chance to win
- You expect to win 50 pence
- $0.35 \times 20 + 0.65 \times (-10) = 7 - 6.5 = 0.5$

Basic Rules

- Arithmetic properties
  - Commutative: $a + b = b + a$, $a \cdot b = b \cdot a$ (or $ab = ba$)
  - Associative: $a(b \cdot c) = (a \cdot b)c = a \cdot b \cdot c$ (or abc)
  - Distributive: $a(b + c) = a \cdot b + a \cdot c$ (or $ab + ac$)

- Fractions
  - Beware inappropriate cancellations
  - $\frac{a+b}{c+b} \neq \frac{a}{c}$ (ex: $\frac{1+2}{3+2} \neq \frac{1}{3}$)
  - Do not break up additive bonds in denominators
  - $\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$ (ex: $\frac{1}{2+3} \neq \frac{1}{2} + \frac{1}{3}$)

Factoring and Expansion

- Factoring
  - Pull common term out of two or more expressions
  - Ex: $ax + \frac{x}{b} = x(a + \frac{1}{b})$
  - Ex: $ax + bx^2 = x(a + bx)$
- Expansion
  - Distribute terms to eliminate parentheses
  - Ex: $x(a + \frac{1}{b}) = ax + \frac{x}{b}$
  - Ex: $x(a + bx) = xa + bx^2$
- FOIL
  - Is $(a + b)^2 = a^2 + b^2$?
  - No, $(a + b)^2 \Rightarrow (a + b)(a + b) \Rightarrow a^2 + 2ab + b^2$

Manipulation of Equations and Inequalities

- Can add (or subtract) any quantity from both sides
- Can multiply (divide) both sides by any (non-zero) quantity
- Sign flips when multiplying/dividing by quantities < 0
- Also flips when rotating inequalities

Use In This Module

- Goal is not to find precise numerical value that satisfies an equation/inequality w/ a single unknown
- Here, we use algebra to generalize
- Will solve for a single variable, but only to establish cutpoints

Cut-point

A critical value, or threshold, above which something different happens than does below.

Example

- Suppose we have $s \geq p(h) + (1-p)(l)$
- Where $0 < l < s < h$ and p is a probability
- For whatever reason, we want to solve for p
- $\Rightarrow s \geq ph + l - pl$
- $\Rightarrow s - l \geq ph - pl$
- $\Rightarrow s - l \geq p(h - l)$
- $\Rightarrow \frac{s-l}{h-l} \geq p$
- $\Rightarrow p \leq \frac{s-l}{h-l}$
- Can say original ineq. holds iff $p \leq \hat{p}$, where $\hat{p} \equiv \frac{s-l}{h-l}$