Game Theory

Essex Summer School

Third Problem Set

Instructions

Problem sets should be submitted either electronically (email to Julian) or in print by the start of class on **Friday, 22nd July**. Mathematical portions may be handwritten, but all verbal explanations should be typed.

1.) Individual i ascribes some intrinsic value to political engagement, ξ_i, where $\xi_i \in [-1, 1]$, and values social acceptance at a rate of $\sigma \in (0, 1]$.[1]

The social network within which i is embedded, n, must decide whether to strengthen ties with i. If n does so, n receives a benefit equal to i's sociability.[2] They also receive $\xi_n \in [-1, 1]$, provided that i is politically engaged. Finally, n always incurs cost $c > 0$ when strengthening ties i.[3]

Imagine that both ξ_i and σ are private information. That is, i knows his or her own values for these terms, but n does not. What n does know is the probability distribution from which these terms are drawn.

[1] That is, we assume that some people enjoy following politics while others do not, and that everyone values being socially accepted though not necessarily to the same degree.

[2] For the sake of simplicity, we'll assume that's the same as their interest in being accepted. In reality, these two qualities are likely distinct even if they are correlated.

[3] This cost simply reflects the finite nature of time and the necessity of investing at least some amount thereof into social ties if they are to have meaning. It may be arbitrarily close to zero. The important thing is that n not have a dominant strategy of accepting.

For simplicity, assume $\xi_i = \overline{\xi}_i$, where $\overline{\xi}_i > 0$, with probability p, and $\xi_i = \underline{\xi}_i$, where $\underline{\xi}_i < 0$, with probability $1 - p$. Further, let $\sigma = \overline{\sigma}$ with probability q and $\sigma = \underline{\sigma}$ with probability $1 - q$, where $\underline{\sigma} < \overline{\sigma}$.[4]

After Nature selects the values of both ξ_i and σ, i decides whether to engage in some form of political behavior that is observable to n, such as discussing current events on Facebook or Twitter. After observing i's decision, n decides whether to strengthen ties. Then the game ends.

If i engages in political behavior and n strengthens ties, i receives $\xi_i + \sigma$ while n receives $\xi_n + \sigma - c$. If i engages in political behavior but n does not strengthen ties, i simply receives ξ_i while n receives 0. If i does not engage in political behavior but n strengthens ties, i receives σ while n receives $\sigma - c$. If i does not engage in political behavior and n does not strengthen ties, both i and n receive 0.

Assume $\xi_n > 0$ Evaluate the possibility of a PBE where i always engages in political behavior if $\xi_i = \overline{\xi}_i$, and only engages in political behavior when $\xi = \underline{\xi}_i$ if $\sigma = \overline{\sigma}$, such that n accepts i iff i engages in political behavior.

[4]Note that p and q are not correlated.