Game Theory
Essex Summer School
Second Problem Set

Instructions

Problem sets should be submitted either electronically (email to me) or in print by the start of class on **Monday, 18th July**. Mathematical portions may be handwritten, but all verbal explanations should be typed.

1.) Find the mixed strategy Nash equilibrium to the following game:

	L	R
U	2, 1	0, 2
D	1, 3	3, 0

2.) A would like B to change some aspect of their behavior. The game begins with A deciding whether to put pressure on B through peaceful means[1] or through violence.[2] Whether A chooses the peace or violence, B must decide whether to give in or resist.[3] Even if B resists, there is nonetheless some chance that they will be forced to make concessions later on down the line.

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[1] Such as protests, if we think of A as a non-state actor, or official statements and/or sit-down meetings involving senior officials if A is a government.

[2] This might take the form of terrorism or insurgency, if A is a non-state actor, or some mix of police and military actions if A is a government

[3] For simplicity, assume that if B gives in, they do so completely (rather than offering partial concessions, which we'll need more advanced models to consider.)

Let $\bar{\rho} = \dfrac{e_A + v}{e_A + e_B + v}$ be the probability that B is forced to make concessions after resisting if A chose violence and let $\underline{\rho} = \dfrac{e_A}{e_A + e_B}$ be the probability that B is forced to make concessions after resisting if A chose peace, where e_A denotes A's efficacy, e_B denotes B's efficacy, and v denotes the utility of violence.[4] That is, we assume that A is always more likely to get what they want if they choose violence than if they do not, though the extent to which that is true may vary and need not be large enough to justify the costs to A.[5] Let c_A denote that cost, and let c_B denote the harm done to B when A resorts to violence. Note that these costs are paid whether B gives in or not.[6]

Finally, assume that there are costs associated with a prolonged standoff, such that A and B incur costs κ_A and κ_B whenever B chooses to resist.[78]

The following table provides the outcomes and utilities.

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[4] All of these terms are assume to be positive.

[5] This assumption is made here purely for the sake of argument. Once you have answered all of the questions below, it should be clear to you what purpose this assumption serves.

[6] We could, of course, assume that A and B both pay greater costs if B does not give in. Doing so, however, would add notational clutter without altering the substantive conclusions I'm trying to get you to see.

[7] We could, of course, introduce separate cost terms for the different types of standoffs. This would be more realistic but would not alter the substantive conclusions.

[8] Note that all cost terms are assumed to be positive. They represent a *cost* in that they are subtracted from the actor's utilities, but their numeric values are greater than zero. We could, alternatively, add negative quantities and achieve the same result, but that would run counter to convention.

Outcomes	u_A	u_B
peace, give in	1	0
peace, resist	$\underline{\rho} - \kappa_A$	$1 - \underline{\rho} - \kappa_B$
violence, give in	$1 - c_A$	$-c_B$
violence, resist	$\overline{\rho} - c_A - \kappa_A$	$1 - \overline{\rho} - c_B - \kappa_B$

Establish a cut-point over κ_B that can be used to determine whether B would resist or give in provided that A chose to pressure them peacefully, which you may call $\hat{\kappa}_p$. Now do the same for violence (I suggest the name $\hat{\kappa}_v$).

Assume $kappa_B$ lies below both of these cut-points, indicating that B will resist regardless of how A chooses to apply pressure. Establish a cut-point over c_A that can be used to determine whether A will choose peace or violence in this particular subset of cases. (I suggest that you call this $\hat{c}_A$.)

Take the partial derivative of the previous cut-point with respect to e_A. What does that tell us, substantively, about the relationship between an actor's ability to get their way and the likelihood that they will resort to violence? If we were to observe, as some have, that those who use violence get what the want less often than those who choose the path of peace, would that be consistent with or contrary to the expectations of this model?