

Game Theory
Essex Summer School
First Problem Set

Instructions

Problem sets should be submitted either electronically (email to [me](#)) or in print by the start of class on **Friday, 15th July**. Mathematical portions may be handwritten, but all verbal explanations should be typed.

1.) Find the derivative of y with respect to x for each of the following

- $y = x^3 - 4x^2 - 6$

- $y = x^{-2}$

- $y = \frac{(x-1)(x+1)}{x+1}$.

2.) Integrate the following derivatives to find y

- $\frac{dy}{dx} = 3x + 3$

- $\frac{dy}{dx} = 5x^2 - x - 4$

- $\frac{dy}{dx} = x^{-2}$.

3.) Identify all extrema (local and global) of the following functions

- $f(x) = 6x - x^2 + 12$, where $x \in [0, 10]$

- $f(x) = 4x^3 + x^2 - 2x + 3$, where $x \in [-1, 1]$.

4.) Suppose some government G would like to both strengthen national security while also improving the economic well-being of its citizens. However, it would like to do so without raising taxes or increasing the national debt. Let B be the amount of resources available if G neither increases taxes nor runs a deficit, let $s \geq 0$ be the amount of resources allocated to national security, and let $w \geq 0$ be the amount devoted to promoting well-being. Finally, let $u_G = 5s - 1.8s^2 + 10w - 0.2w^2$, indicating that there are diminishing marginal returns—as well as increasing marginal costs—associated with public spending. Identify the optimal s and w subject to the constraint $s + w \leq B$.