Online Appendix

Before proceeding to the proofs of the Lemmas and Propositions, I first derive some key cutpoints that will be useful for characterizing the equilibria of interest. The following serves to fully characterize all wartime decisions made by G and O, which are necessary to characterize G's acceptance rule, F's optimal proposal, and O's initial position.

Begin with G's decision to quit rather than continue. Generically, if O advocated "p" at the outset, then G quits iff $u_G(\text{quit}) \geq u_G(\text{continue})$, which holds when

$$\beta w_s - c_D \geq \beta w_l - \kappa_D,$$

or

$$\kappa_D \geq \beta(w_l - w_s) + c_D.$$

Then we can then say that if O advocates "p", should G reject F's terms, during the resulting war, G will quit iff $\kappa_D \leq \kappa_D^p$, where $\kappa_D^p \equiv \beta(w_l - w_s) + c_D$. That is, should there be a war, it will be a short war if $\kappa_D < \kappa_D^p$, and a long war if $\kappa_D \geq \kappa_D^p$.

This means that, if O initially advocated "p", G quits when relatively high in resolve if and only if $\underline{\kappa}_D \geq \underline{\kappa}_D^p$, where $\underline{\kappa}_D^p \equiv \beta(w_l - w_s) + \underline{c}_D$, and if and only if $\overline{\kappa}_D \geq \overline{\kappa}_D^p$, where $\overline{\kappa}_D^p \equiv \beta(w_l - w_s) + \overline{c}_D$, when relatively low in resolve.

When O advocates "w" followed by "q", then $u_G(\text{quit}) \geq u_G(\text{continue})$ holds iff

$$w_s - c_D \geq \beta w_l - \kappa_D,$$

or

$$\kappa_D \geq \beta w_l - w_s + c_D.$$

Then we can then say that if O advocates "w" then "q", should G reject F's terms, during the resulting war, G will quit iff $\kappa_D \leq \kappa_D^{w,q}$, where $\kappa_D^{w,q} \equiv \beta w_l - w_s + c_D$.

This means that, if O initially advocated "w" then eventually advocates "q", G quits if and only if $\underline{\kappa}_D \geq \underline{\kappa}_D^{w,q}$, where $\underline{\kappa}_D^{w,q} \equiv \beta w_l - w_s + \underline{c}_D$, when relatively resolved, and quits if and only if $\overline{\kappa}_D \geq \overline{\kappa}_D^{w,q}$, where $\overline{\kappa}_D^{w,q} \equiv \beta w_l - w_s + \overline{c}_D$, when relatively low in resolve.

When O advocates "w" followed by "c", then $u_G(\text{quit}) \geq u_G(\text{continue})$ holds iff

$$\beta w_s - c_D \geq w_l - \kappa_D,$$

or

$$\kappa_D \geq w_l - \beta w_s + c_D.$$

Then we can then say that if O advocates "w" then "c", should G reject F's terms, during the resulting war, G will quit iff $\kappa_D \leq \kappa_D^{w,c}$, where $\kappa_D^{w,q} \equiv w_l - \beta w_s + c_D$.

This means that, if O initially advocated "w" then eventually advocates "c", G quits if and only if $\underline{\kappa}_D \geq \underline{\kappa}_D^{w,c}$, where $\underline{\kappa}_D^{w,c} \equiv w_l - \beta w_s + \underline{c}_D$, when relatively resolved, and quits if and only if $\overline{\kappa}_D \geq \overline{\kappa}_D^{w,c}$, where $\overline{\kappa}_D^{w,c} \equiv w_l - \beta w_s + \overline{c}_D$, when relatively low in resolve.

When κ_D takes on sufficiently high or low values, O's strategy follows directly from w_s and w_l, since G's decision will not depend on O's position, and so O need only determine which position will put them on the right side of history.

Generically, if $\kappa_D > max\{\kappa_D^{w,q}, \kappa_D^{w,c}\}$, the war is going to be a short war no matter what O says, since G will quit either way, and $u_O(\text{"q"}) \geq u_O(\text{"c"})$ holds iff

$$w_s - c_D \geq \rho w_s - c_D,$$

or iff $w_s \geq 0$.

When $\kappa_D < min\{\kappa_D^{w,q}, \kappa_D^{w,c}\}$, G is sure to continue, and $u_O(\text{"q"}) \geq u_O(\text{"c"})$ holds iff

$$\rho w_l - \kappa_D \geq w_l - \kappa_D,$$

or iff $w_l < 0$.

In contrast, when $\kappa_D^{w,q} \leq \kappa_D < \kappa_D^{w,c}$, O advocates "q" iff

$$w_s - c_D \geq w_l - \kappa_D,$$

or

$$\kappa_D \geq w_l - w_s + c_D.$$

And when $\kappa_D^{w,c} \leq \kappa_D < \kappa_D^{w,q}$, O advocates "q" iff

$$\rho w_l - \kappa_D \geq \rho w_s - c_D,$$

or

$$\rho(w_l - w_s) \geq \kappa_D - c_D.$$

Note that, depending on the signs on w_l and w_s, any ordering between the three cutpoints over κ_D is possible. However, when $w_s < 0$ and $w_l < 0$, it must be true that $\kappa_D^{w,q} < \kappa_D^{p} < \kappa_D^{w,c}$. As discussed in the paper, that is the most interesting case, and thus will largely be the focus of my attention in the remainder of the appendix.

I now turn to proofs of the Lemmas and Propositions discussed in the text.

Proofs of the Lemmas

Lemma 1. I first discuss the conditions under which the bargaining range is empty.

Let $\overline{x}_F$ denote a cutpoint such that $U_F(peace) \geq U_F(war)\ \forall\ x \leq \overline{x}_F$ and $U_F(peace) < U_F(war)\ \forall\ x > \overline{x}_F$. That is, let $\overline{x}_F$ be the best terms that F is willing to offer G.

Let $\underline{x}_G$ denote a cutpoint such that $U_G(peace) \geq U_G(war)\ \forall\ x \geq \underline{x}_G$ and $U_G(peace) < U_G(war)\ \forall\ x < \underline{x}_G$. In words, $\underline{x}_G$ is the worst offer from F that G is willing to accept.

It immediately follows that both players prefer a peaceful agreement to war when $x \in [\underline{x}_G, \overline{x}_F]$. The set of agreements that F and G mutually prefer to war is empty when $\underline{x}_G > \overline{x}_F$. Put differently, if $\underline{x}_G > \overline{x}_F$, F is willing to offer terms that are more attractive than the worst terms that G would accept. Thus, a bargaining range exists. In contrast, the bargaining range is empty when $\underline{x}_G > \overline{x}_F$, as the best terms that F is willing to offer are still less attractive to G than the worst terms that G would ever accept. As a result, war is certain to occur, regardless of what F believes about the democratic state's resolve.

Per the Lemma, assume that $w_s > 0$ and $w_l > 0$, indicating that the course of fighting would favor the democratic state.

If O advocates "w" and G accepts F's proposal of x, G will receive βx. Should G instead reject, G will receive either $w_s - c_D$, $\beta w_s - c_D$, $w_l - \kappa_D$ or $\beta w_l - \kappa_D$, depending upon the optimal wartime strategies for G and O. Suppose $\kappa_D \geq \kappa_D^{w,q}$, which ensures that if G rejects F's terms, O will advocate quitting and G will do so. In this case, $u_G(\text{peace}) \geq u_G(\text{short war})$ holds generically iff

$$\beta x \geq w_s - c_D,$$

or

$$x \geq \frac{w_s - c_D}{\beta}.$$

Thus $\underline{x}_G = \dfrac{w_s - c_D}{\beta}$.

Setting aside for the moment the question of whether G will accept, $u_F(\text{peace}) \geq u_F(\text{short war})$ is certain to hold when O advocates "w" and $\kappa_D \geq \kappa_D^{w,q}$ iff

$$-x \geq -w_s - c_F,$$

and thus $\overline{x}_F = w_s + c_F$.

Using the above definition, a bargaining range exists iff $\underline{x}_G \leq \overline{x}_F$, or

$$\frac{w_s - c_D}{\beta} \leq w_s + c_F,$$

which is equivalent to

$$c_F + \frac{c_D}{\beta} \geq w_s(\frac{1}{\beta} - 1).$$

Since we have already assumed $w_s > 0$, this inequality is trivially satisfied for any non-negative values of any of the cost terms, which are strictly positive by assumption.

Now suppose instead that O initially advocated "w" and $\kappa_D < \kappa_D^{w,c}$, ensuring that O will advocate "c" and G will indeed continue the war. Then $u_G(\text{peace}) \geq u_G(\text{long war})$ holds generically iff

$$\beta x \geq w_l - \kappa_D,$$

or

$$x \geq \frac{w_l - \kappa_D}{\beta}.$$

Thus $\underline{x}_G = \dfrac{w_l - \kappa_D}{\beta}$.

Here, $u_F(\text{peace}) \geq u_F(\text{short war})$ holds iff

$$-x \geq -w_l - \kappa_F,$$

and thus $\overline{x}_F = w_l + \kappa_F$.

Here, a bargaining range exists iff $\underline{x}_G \leq \overline{x}_F$, or

$$\frac{w_l - \kappa_D}{\beta} \leq w_l + \kappa_F,$$

which is equivalent to

$$\kappa_F + \frac{\kappa_D}{\beta} \geq w_l(\frac{1}{\beta} - 1).$$

Again, since we have already assumed $w_l > 0$, this inequality is trivially satisfied.

Next, suppose that O initially advocated "w" and $\kappa_D^{w,c} \leq \kappa < \kappa_D^{w,c}$. In such cases, were G to reject F's terms, G would quit if and only if O advocated continuing. O will indeed advocate continuing if $\rho(w_l - w_s) \geq \kappa_D - c_D$, as discussed above. Should this inequality hold, then $u_G(\text{peace}) \geq u_G(\text{long war})$ holds generically iff

$$\beta x \geq \beta w_s - c_D,$$

or

$$x \geq w_s - \frac{c_D}{\beta}.$$

Thus $\underline{x}_G = w_s - \dfrac{c_D}{\beta}$.

Since F's payoff does not depend upon whether the war is politicized, $\overline{x}_F$ here is $w_s + c_F$, as it was when $\kappa_D \geq \kappa_D^{w,q}$.

Thus, a bargaining range exists iff $\underline{x}_G \leq \overline{x}_F$, or

$$w_s - \frac{c_D}{\beta} \leq w_s + c_F,$$

which is equivalent to

$$c_F + \frac{c_D}{\beta} \geq 0,$$

which must be true.

Suppose instead that O initially advocated "w", $\kappa_D^{w,c} \leq \kappa < \kappa_D^{w,c}$, and $\rho(w_l - w_s) < \kappa_D - c_D$, indicating that, should G reject F's terms, there will a long, politicized war. Then $u_G(\text{peace}) \geq u_G(\text{long war})$ holds generically iff

$$\beta x \geq \beta w_l - \kappa_D,$$

or

$$x \geq w_l - \frac{\kappa_D}{\beta}.$$

Thus $\underline{x}_G = w_l - \dfrac{\kappa_D}{\beta}$.

Again, F's payoff does not depend upon whether the war is politicized, so $\overline{x}_F$ is $w_l + \kappa_F$, as it was when $\kappa_D < \kappa_D^{w,c}$.

Thus, a bargaining range exists iff $\underline{x}_G \leq \overline{x}_F$, or

$$w_l - \frac{\kappa_D}{\beta} \leq w_l + \kappa_F,$$

which is equivalent to

$$\kappa_F + \frac{\kappa_D}{\beta} \geq 0.$$

Thus, if the war is expected to favor the democratic state, a bargaining range must exist if O advocates “w”, per the Lemma.

Finally, suppose O advocates “p”. If G accepts, G will receive βx. Should G instead reject, G will receive either $\beta w_s - c_D$ or $\beta w_l - \kappa_D$, depending upon whether κ_D merits fighting a long war or a short war.

Suppose first that $\kappa_D \geq \kappa_D^p$, which ensures that if G rejects F's terms, G will only fight a short war. Then $u_G(\text{peace}) \geq u_G(\text{short war})$ holds generically iff

$$x \geq \beta w_s - c_D.$$

Since F does not care whether outcomes are politicized, $\overline{x}_F = w_s + c_F$.

Here, a bargaining range exists iff $\underline{x}_G \leq \overline{x}_F$, or

$$\beta w_s - c_D \leq w_s + c_F,$$

which is equivalent to

$$c_F + \frac{c_D}{\beta} \geq w_s(1 - \frac{1}{\beta}).$$

Since both sides of the inequality are strictly positive, the bargaining range may be empty.

It is straightforward to establish that if O advocates “p” and $\kappa_D < \kappa_D^p$, we need only replace w_s with w_l, c_D with κ_D, and c_F with κ_F. Thus, war is inefficient iff

$$\kappa_F + \frac{\kappa_D}{\beta} \geq w_l(1 - \frac{1}{\beta}).$$

Again, since both sides are strictly positive, the bargaining range may be empty when O advocates “p”, provided the course of fighting would favor the democratic state.

This establishes the result. $\square$

Lemma 2. Return to the proof for Lemma 1.

A bargaining range was found to exist when O advocates "p" provided that

$$c_F + \frac{c_D}{\beta} \geq w_s(1 - \frac{1}{\beta}),$$

when $\kappa_D \geq \kappa_D^p$, and when

$$\kappa_F + \frac{\kappa_D}{\beta} \geq w_l(1 - \frac{1}{\beta}).$$

when $\kappa_D < \kappa_D^p$.

If the course of fighting is expected to favor the foreign state (i.e., if $w_s < 0$ and $w_l < 0$), the right hand side of each of these inequalities is negative. They are thus trivially satisfied, since the cost terms are positive by assumption.

When O advocates "w", a bargaining range exists provided that

$$c_F + \frac{c_D}{\beta} \geq w_s(\frac{1}{\beta} - 1),$$

if $\kappa_D \geq \kappa_D^{w,q}$, and provided that

$$\kappa_F + \frac{\kappa_D}{\beta} \geq w_l(\frac{1}{\beta} - 1).$$

if $\kappa_D < \kappa_D^{w,c}$.

When $w_s < 0$ and $w_l < 0$, the right hand side of these inequalities is strictly positive.

Note that when $\kappa_D^{w,c} \leq \kappa_D < \kappa_D^{w,q}$, a bargaining range exists so long as the costs of war are positive, which they are by assumption. Nonetheless, for sufficiently large or sufficiently small values of κ_D, the bargaining range may be empty when O advocates "w" and the course of fighting would favor the foreign state.

This establishes the result. □

Proofs of the Propositions

Proposition 1. If a bargaining range exists, F proposes values of x that G accepts with positive probability. This is what is meant in the text as bargaining in good faith. Note that war may still occur, since F may find it optimal to propose values of x that G accepts if and only if the democratic state is relatively low in resolve. But this is still distinct from cases where the bargaining range is empty and the *ex ante* probability of war is thus 1.

When bargaining in good faith, F has no incentive to set x greater than the value G accepts when relatively high in resolve, as such proposals are already certain to be accepted, and u_F(peace) is strictly decreasing in x. Further, F has no incentive to set x less than the value G accepts when relatively low in resolve, since such proposals provoke war with certainty. When a bargaining range exists, F must prefer to make proposals that are acceptable to at least one type. Finally, F has no incentive to set x strictly between these two values, since doing so entails F receiving their war payoff when G is relatively resolved and receiving worse terms when G is relatively unresolved compared to what F would have received had F set x to the less resolved type's reservation value.

This proof has two parts.

First, suppose $\overline{\kappa}_D \geq \{\overline{\kappa}^p_D, \overline{\kappa}^{w,q}_D, \overline{\kappa}^{w,c}_D\}$, $c_F + \overline{c}_D \geq w_s(\beta - 1)$, $\underline{\kappa}_D \geq \{\underline{\kappa}^p_D, \underline{\kappa}^{w,q}_D, \underline{\kappa}^{w,c}_D\}$, $c_F + \underline{c}_D \geq w_s(\beta - 1)$ and $w_s > 0$. This ensures that, should G reject x, G will quit regardless of the democratic state's level of resolve, and regardless of O's strategy. It also ensures that F bargains in good faith regardless of O's initial statement.

If O advocates "p", then $\underline{x}_G = \beta w_s - c_D$, since we have already stipulated $\kappa_D \geq \kappa^p_D$ for both the relatively resolved and less resolved types, ensuring that G will fight a short war if G fights a war at all.

We have also already stipulated values that ensure that a bargaining range exists. Thus F must prefer to either set $x = \beta w_s - \underline{c}_D$, which is accepted with certainty but yields a relatively low payoff for F, or to set $x = \beta w_s - \overline{c}_D$, which is accepted only if the democratic state has relatively low resolve but leaves F better off when it is in fact accepted.

F prefers to play it safe by choosing the former terms so long as $u_F(x = \beta w_s - \underline{c}_D) \geq E(u_F(x = \beta w_s - \bar{c}_D))$, which holds when

$$-(\beta w_s - \underline{c}_D) \geq \phi'_p(-(\beta w_s - \bar{c}_D)) + (1 - \phi'_p)(-w_s - c_F),$$

where ϕ'_p is the updated value of ϕ given that O advocated "p".

If we solve this for ϕ'_p, we obtain

$$\phi'_p \leq \frac{w_s(1-\beta) + c_F + \underline{c}_D}{w_s(1-\beta) + c_F + \bar{c}_D}.$$

We can thus say that F sets $x = \beta w_s - \underline{c}_D$ iff $\phi'_p \leq \hat{\phi}^p_q$, where $\hat{\phi}^p_q \equiv \dfrac{w_s(1-\beta) + c_F + \underline{c}_D}{w_s(1-\beta) + c_F + \bar{c}_D}$.

If O advocates "w", then $\underline{x}_G = \dfrac{w_s - c_D}{\beta}$, since we have stipulated values that ensure that G will quit, and O will advocate quitting, if G fights at all.

Given this, F prefers to play it safe by setting $x = \dfrac{w_s - \underline{c}_D}{\beta}$ rather than risking war by setting $x = \dfrac{w_s - \bar{c}_D}{\beta}$ iff $u_F(x = \dfrac{w_s - \underline{c}_D}{\beta}) \geq E(u_F(x = \dfrac{w_s - \bar{c}_D}{\beta}))$, which holds so long as

$$-(\frac{w_s - \underline{c}_D}{\beta}) \geq \phi'_w(-(\frac{w_s - \bar{c}_D}{\beta})) + (1 - \phi'_w)(-w_s - c_F),$$

where ϕ'_w is the updated value of ϕ given that O advocated "w".

We again solve for ϕ'_w, obtaining

$$\phi'_w \leq \frac{w_s(1 - \frac{1}{\beta}) + c_F + \frac{\underline{c}_D}{\beta}}{w_s(1 - \frac{1}{\beta}) + c_F + \frac{\underline{c}_D}{\beta}}.$$

We can thus say that F sets $x = \dfrac{w_s - \underline{c}_D}{\beta}$, which is accepted with certainty, iff $\phi'_w \leq \hat{\phi}^{w,q}_q$, where $\hat{\phi}^{w,q}_q \equiv \dfrac{w_s(1 - \frac{1}{\beta}) + c_F + \frac{\underline{c}_D}{\beta}}{w_s(1 - \frac{1}{\beta}) + c_F + \frac{\underline{c}_D}{\beta}}$.

We are interested in a separating perfect Bayesian equilibrium where O advocates "p" if the democratic state is relatively less resolved and O advocates "w" if the democratic state is relatively resolved. In such an equilibrium, by Bayes' rule, $\phi'_p = 1$ and $\phi'_w = 0$.

This naturally implies $\phi'_p > \hat{\phi}^p_q$ and $\phi'_w \leq \hat{\phi}^{w,q}_q$, since both $\hat{\phi}^p_q$ and $\hat{\phi}^{w,q}_q$ lie in the interval $(0, 1)$. Thus, in such an equilibrium, F sets $x = \beta w_s - \overline{c}_D$ if O advocates "p" and sets $x = \dfrac{w_s - \underline{c}_D}{\beta}$ if O advocates "w".

When the democratic state is relatively less resolved, G will accept either offer. Incentive for compatibility for O is therefore equivalent to

$$\beta w_s - \overline{c}_D \geq \rho(\frac{w_s - \underline{c}_D}{\beta}),$$

or

$$w_s(\beta - \frac{\rho}{\beta}) \geq \overline{c}_D - \frac{\rho}{\beta}\underline{c}_D.$$

When relatively resolved, incentive compatibility for O requires that the payoff to O when G accepts $x = \dfrac{w_s - \underline{c}_D}{\beta}$ be at least as good as O's payoff when G rejects x and a short war results. This is satisfied so long as

$$\rho(\frac{w_s - \underline{c}_D}{\beta}) \geq \rho w_s - \underline{c}_D,$$

or

$$w_s(\rho - \frac{\rho}{\beta}) \leq \underline{c}_D(1 - \frac{\rho}{\beta}).$$

For each of the incentive compatibility constraints, both the left hand side and the right hand side of the inequality are strictly positive. Thus, the separating equilibrium of interest must exist for some range of values when $w_s > 0$.

Now suppose $w_s < 0$, but keep all the other conditions the same as above.

If O advocates "w", G will fight a short war if G fights at all, regardless of O's behavior. O will advocate "c", since $\rho w_s - c_D$ is strictly greater than $w_s - c_D$ when $w_s < 0$. Therefore, when O advocates "w", $\underline{x}_G = w_s - \frac{c_D}{\beta}$, and a bargaining range exists. Note that $w_s < 0$ ensures the existence of a bargaining range when O advocates "p".

As above, when O advocates "p", F's optimal strategy is to choose $x = \beta w_s - \underline{c}_D$ iff $\phi'_p \leq \hat{\phi}^p_q$, otherwise setting $x = \beta w_s - \overline{c}_D$.

If O advocates "w", F plays it safe by setting $x = w_s - \frac{\underline{c}_D}{\beta}$ rather than risking war by setting $x = w_s - \frac{\overline{c}_D}{\beta}$ iff $u_F(x = w_s - \frac{\underline{c}_D}{\beta}) \geq E(u_F(x = w_s - \frac{\overline{c}_D}{\beta}))$, which holds so long as

$$-(w_s - \frac{\underline{c}_D}{\beta}) \geq \phi'_w(-(-w_s - \frac{\overline{c}_D}{\beta})) + (1 - \phi'_w)(-w_s - c_F).$$

We again solve for ϕ'_w, obtaining

$$\phi'_w \leq \frac{c_F + \frac{\underline{c}_D}{\beta}}{c_F + \frac{\overline{c}_D}{\beta}}.$$

We can thus say that F sets $x = w_s - \frac{\underline{c}_D}{\beta}$, which is accepted by G with certainty, rather than $x = w_s - \frac{\overline{c}_D}{\beta}$, which risks war, iff $\phi'_w \leq \hat{\phi}^{w,c}_q$, where $\hat{\phi}^{w,c}_q \equiv \frac{c_F + \frac{\underline{c}_D}{\beta}}{c_F + \frac{\overline{c}_D}{\beta}}$.

Again, we are interested in a separating equilibrium where O advocates "p" if relatively low in resolve and "w" if relatively high in resolve, and thus $\phi'_p = 1 > \hat{\phi}^p_q$ and $\phi'_w = 0 < \hat{\phi}^{w,c}_q$.

When the democratic state is low in resolve, O's incentive compatibility constraint requires that O be better off with G accepting $x = \beta w_s - \overline{c}_D$, as G will if O advocates "p", then O would be with G accepting $w_s - \frac{\underline{c}_D}{\beta}$, which G will do if O advocates "w".

This is satisfied when $\beta w_s - \overline{c}_D \geq \rho(w_s - \frac{\underline{c}_D}{\beta})$, which cannot be true, given that $w_s < 0$. Thus the equilibrium fails. □

Proposition 2. The existence of a PBE of the following form proves the result:

- O advocates "w" then "q", regardless of type.
- After O advocates "w", G accepts any $x \geq \frac{w_s - \underline{c}_D}{\beta}$, rejecting otherwise, when relatively resolved, and accepts any $x \geq \frac{w_s - \overline{c}_D}{\beta}$, rejecting otherwise, when relatively less resolved and quits iff O advocates "q", while after O advocates "p", G accepts any $x \geq \beta w_s - \underline{c}_D$, rejecting otherwise, when relatively resolved, and accepts any $x \geq \beta w_s - \overline{c}_D$, rejecting otherwise, when less resolved, and quits.
- After O advocates "w", F sets $x < \frac{w_s - \underline{c}_D}{\beta}\ \forall\ \phi'_w$, while after O advocates "p", F sets $x = \beta w_s - \underline{c}_D$ if $\phi'_p \leq \hat{\phi}^p_q$ and $x = \beta w_s - \overline{c}_D$ if $\phi'_p > \hat{\phi}^p_q$.

Suppose $w_s < 0$ and $w_l < 0$, $\overline{\kappa}^p_D \leq \overline{\kappa}_D < \overline{\kappa}^{w,c}_D$ and $\underline{\kappa}^p_D \leq \underline{\kappa}_D < \underline{\kappa}^{w,c}_D$. Note that $\overline{\kappa}_D \geq \overline{\kappa}^{w,q}_D$ and $\underline{\kappa}_D \geq \underline{\kappa}^{w,q}_D$ are then trivially satisfied, since $\kappa^p_D > \kappa^{w,q}_D$ must hold when $w_s < 0$ and $w_l < 0$. Given these conditions, the above strategies for G are optimal.

Next suppose $\overline{\kappa}_D \geq w_l - w_s + \overline{c}_D$ and $\underline{\kappa}_D \geq w_l - w_s + \underline{c}_D$. This ensures that O's strategy after G rejects x is optimal, regardless of the democratic state's resolve.

Now let $c_F + \frac{\overline{c}_D}{\beta} < w_s(\frac{1}{\beta} - 1)$. Then the above strategies for F after O advocates "w" and "p" follow from the proofs of Lemma 2 and Proposition 1, respectively.

All that remains is the incentive compatibility constraints for O.

The incentive compatibility constraints depend on the off-the-equilibrium-path belief held by F. Since this belief cannot be determined via Bayes' Rule, I will consider all possibilities.

First, if $\phi'_p \leq \hat{\phi}^p_q$, incentive compatibility for the less resolved type requires that O's payoff when G fights a short, non-politicized war be at least as good as O's payoff from having G accept $x = \beta w_s - \underline{c}_D$. This holds so long as

$$w_s - \overline{c}_D \geq \beta w_s - \underline{c}_D,$$

or

$$w_s(1-\beta) \geq \overline{c}_D - \underline{c}_D.$$

For the more resolved type, incentive compatibility requires that O's payoff when G fights a short, non-politicized, war be at least as good as O's payoff from having G accept $x = \beta w_s - \underline{c}_D$. This holds so long as

$$w_s - \underline{c}_D \geq \beta w_s - \underline{c}_D,$$

which simplifies to $w_s(1-\beta) \geq 0$, which must be true, given $w_s < 0$.

Now suppose that $\phi_p' > \hat{\phi}_q^p$.

Here, incentive compatibility for the less resolved type requires that O's payoff when G fights a short war that O will have initially advocated fighting and later advocated quitting be at least as good as O's payoff from having G accept $x = \beta w_s - \overline{c}_D$. This holds so long as

$$w_s - \overline{c}_D \geq \beta w_s - \overline{c}_D,$$

which simplifies to $w_s(1-\beta) \geq 0$, which must be true, given $w_s < 0$.

For the more resolved type, incentive compatibility requires that O's payoff when G fights a short war that O will have initially advocated fighting and later advocated quitting be at least as good as O's payoff from having G reject $x = \beta w_s - \overline{c}_D$ and fighting a short, politicized, war. This holds so long as

$$w_s - \underline{c}_D \geq \rho w_s - \underline{c}_D,$$

which simplifies to $w_s(1-\rho) \geq 0$, which cannot be true.

Thus, the equilibrium of interest is possible, provided $\phi_p' \leq \hat{\phi}_q^p$. □

Proposition 3. The existence of the following PBE proves the result:

- O advocates "w" then "q", regardless of type.
- After O advocates "w", G accepts any $x \geq w_l - \frac{\underline{\kappa}_D}{\beta}$, rejecting otherwise, when relatively resolved, and accepts any $x \geq w_l - \frac{\overline{\kappa}_D}{\beta}$, rejecting otherwise, when relatively less resolved, and continues fighting regardless of O's wartime position. After O advocates "p", G accepts any $x \geq \beta w_l - \underline{\kappa}_D$, rejecting otherwise, when relatively resolved, and accepts any $x \geq \beta w_l - \overline{\kappa}_D$, rejecting otherwise, when relatively less resolved, and continues fighting.
- F sets $x = w_l - \frac{\overline{\kappa}_D}{\beta}$ if O advocates "w", and $x = \beta w_l - \underline{\kappa}_D$ if O advocates "p".

Suppose $w_s < 0$ and $w_l < 0$, $\overline{\kappa}_D < \overline{\kappa}_D^{w,q}$ and $\underline{\kappa}_D < \underline{\kappa}_D^{w,q}$. Note that $\overline{\kappa}_D < \overline{\kappa}_D^{w,c}$, $\overline{\kappa}_D < \overline{\kappa}_D^{p}$, $\underline{\kappa}_D < \underline{\kappa}_D^{w,c}$ and $\underline{\kappa}_D < \underline{\kappa}_D^{p}$ are trivially satisfied, since $\kappa_D^{w,c} > \kappa_D^{w,q}$ and $\kappa_D^{p} > \kappa_D^{w,q}$ must hold given the assumption that $w_s < 0$ and $w_l < 0$. These conditions ensure that the strategies for G outlined above are optimal.

That O must strictly prefer to advocate "q" after advocating "w" follows trivially from G's strategies and $w_l < 0$, since $\rho w_l - \kappa_D > w_l - \kappa_D$.

When O advocates "p", F prefers to play it safe by setting $x = \beta w_l - \underline{\kappa}_D$, which G is sure to accept, rather than risking war by setting $x = \beta w_l - \overline{\kappa}_D$, which G accepts only if the democratic state is relatively low in resolve, iff $u_F(x = \beta w_l - \underline{\kappa}_D) \geq E(u_F(x = \beta w_l - \overline{\kappa}_D))$, which holds when

$$-(\beta w_l - \underline{\kappa}_D) \geq \phi_p'(-(\beta w_l - \overline{\kappa}_D)) + (1 - \phi_p')(-w_l - \kappa_F).$$

If we solve this for ϕ_p', we obtain

$$\phi_p' \leq \frac{w_l(1-\beta) + \kappa_F + \underline{\kappa}_D}{w_l(1-\beta) + \kappa_F + \overline{\kappa}_D}.$$

We can thus say that F sets $x = \beta w_l - \underline{\kappa}_D$, per the equilibrium, so long as $\phi'_p \leq \hat{\phi}^p_c$, where $\hat{\phi}^p_c \equiv \dfrac{w_l(1-\beta) + \kappa_F + \underline{\kappa}_D}{w_l(1-\beta) + \kappa_F + \overline{\kappa}_D}$.

When O advocates "w", F prefers to play it safe by setting $x = w_l - \dfrac{\underline{\kappa}_D}{\beta}$, which G is sure to accept, rather than risking war by setting $x = w_l - \dfrac{\overline{\kappa}_D}{\beta}$, which G accepts only if the democratic state is relatively low in resolve, iff $u_F(x = w_l - \dfrac{\underline{\kappa}_D}{\beta}) \geq E(u_F(x = w_l - \dfrac{\overline{\kappa}_D}{\beta}))$, which holds when

$$-(w_l - \frac{\underline{\kappa}_D}{\beta}) \geq \phi'_w(-(w_l - \frac{\overline{\kappa}_D}{\beta})) + (1-\phi'_w)(-w_l - \kappa_F).$$

If we solve this for ϕ'_p, we obtain

$$\phi'_w \leq \frac{\kappa_F + \frac{\underline{\kappa}_D}{\beta}}{\kappa_F + \frac{\overline{\kappa}_D}{\beta}}.$$

We can thus say that F sets $x = w_l - \dfrac{\underline{\kappa}_D}{\beta}$ only when $\phi'_w \leq \hat{\phi}^{w,q}_c$, where $\hat{\phi}^{w,q}_c \equiv \dfrac{\kappa_F + \frac{\underline{\kappa}_D}{\beta}}{\kappa_F + \frac{\overline{\kappa}_D}{\beta}}$. Provided $\phi'_w > \hat{\phi}^{w,q}_c$, F will set $x = w_l - \dfrac{\overline{\kappa}_D}{\beta}$, per the equilibrium.

Thus, the strategies outlined for F above are sequentially rational provided that $\phi'_p \leq \hat{\phi}^p_c$ and $\phi'_w > \hat{\phi}^{w,q}_c$. Since this is a pooling equilibrium, F does not update, and ϕ'_p is an off-the-equilibrium-path belief while ϕ'_w will simply equal ϕ.

Note that the equilibrium can exist without the off-the-equilibrium-path belief taking on strange values. If this belief were also equal to ϕ, indicating that F makes no assumptions about either type being more likely to deviate from the equilibrium strategy, then so long as $\hat{\phi}^{w,q}_c < \phi < \hat{\phi}^p_c$, F's strategy would be sequentially rational.

Suppose, as in the example given in the text, that $w_l = -0.2$, $\beta = 2$, $\underline{\kappa}_D = 0.2$, $\overline{\kappa}_D = 0.5$. Suppose further that $\kappa_F = 0.1$ and $\phi = 0.6$. Then $\hat{\phi}^p_c = 0.625$, while $\hat{\phi}^{w,q}_c$ would be approximately 0.57, and it would thus be true that $\hat{\phi}^p_c < \phi < \hat{\phi}^{w,q}_c$.

This leaves only O's incentive compatibility constraints.

For the less resolved type, incentive compatibility requires that O's payoff when G accepts $x = w_l - \frac{\overline{\kappa}_D}{\beta}$, as G will if O advocates "w", be at least as good as O's payoff when G accepts $x = \beta w_l - \underline{\kappa}_D$, as G will if O advocates "p". This holds so long as

$$\rho(w_l - \frac{\overline{\kappa}_D}{\beta}) \geq \beta w_l - \underline{\kappa}_D,$$

or

$$w_l(\rho - \beta) \geq \frac{\rho}{\beta}\overline{\kappa}_D - \underline{\kappa}_D.$$

Note that, given $w_l < 0$, the left hand side is strictly positive, while the right hand side is potentially negative, depending upon the size of ρ and β.

For the more resolved type, incentive compatibility requires that O's payoff when G rejects $x = w_l - \frac{\overline{\kappa}_D}{\beta}$ and fights a long war that will ultimately become politicized, as will occur if O advocates "w", be at least as good as O's payoff when G accepts $x = \beta w_l - \underline{\kappa}_D$, which results when O advocates "p". This holds so long as

$$\rho w_l - \underline{\kappa}_D \geq \beta w_l - \underline{\kappa}_D.$$

Given $w_l < 0$, this simplifies to $\rho \leq \beta$, which must be true.

Therefore, the incentive compatibility conditions for both types are satisfied provided $w_l(\rho - \beta) \geq \frac{\rho}{\beta}\overline{\kappa}_D - \underline{\kappa}_D$. This completes the proof. □