Game Theory

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Bargaining

Introduction

- Three goals for this session
  1. Explain Nash bargaining solution
  2. Analyze Rubinstein bargaining model
  3. Application: crisis bargaining

Basic Terminology

Definition (Bargaining Situation)

Any instance where two or more parties stand to benefit from reaching an agreement (which may be tacit), but have conflicting interests over the division of the benefits.

Definition (Bargaining Model)

A formal model in which one or more actors determines how the disputed good(s) would be divided in the event of an agreement.

Nash's General Bargaining Game

- There is some good π to be divided b/w A and B
- Let x_i denote i's share, implying $x_A + x_B = \pi$
- If no agreement, A receives d_A and B receives d_B
- By assumption, $\exists$ (x_A, x_B) $\ni$ $u_i(x_i) \geq u_i(d_i)$ $\forall$ $i \in \{A, B\}$
- Let $v_i = u_i(x_i)$

The Nash Bargaining Solution

- The Nash Bargaining Solution is often denoted (v_A^N, v_B^N)
- It is the solution to the following: $\max_{x_A}(v_A - d_A)(v_B - d_B)$
- Let $d_i = 0$ and $u_i(x_i) = v_i = x_i, \; \forall \; i \in \{A, B\}$
- $\max_{x_A}((x_A)(x_B)) = \max_{x_A}((x_A)(\pi - x_A))$
- $\max_{x_A}(x_A\pi - x_A^2)$
- $\pi - 2x_A = 0$
- $x_A^N = \frac{\pi}{2}$

Implications

- Iff both players derive equal utility from each unit increase in the good and have equal disagreement values, the Nash Bargaining Solution (NBS) will be an equal division
- When $u_i(x_i) = v_i = x_i$, $v_A^N = \frac{1}{2}(\pi - d_B + d_A)$
- Leaving B with $x_B^N = \pi - x_A^N$
- The easier it is to walk away, the larger one's share

Rubinstein Bargaining Model

- A and B are negotiating over some good worth 1 to each
- Suppose neither has credible outside option
- Neither enjoys disagreeing either, but costs unequal
- Game continues until agreement reached
- If agreement reached in t, A receives x_t and B receives $1 - x_t$
- A and B take turns proposing divisions
- Future payoffs discounted by $\delta_i \; \forall \; i \in \{A, B\}$

Analysis

- Rubinstein identified a unique SPNE
- A sets $x = x_1^R$, where $x_1^R \equiv \frac{1-\delta_B}{1-\delta_A\delta_B}$ in all odd periods and accepts B's counteroffers in even periods iff $x \geq \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}$
- B sets $x = x_2^R$, where $x_2^R \equiv \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}$ in all even periods and accepts A's offers in odd periods iff $x \leq \frac{1-\delta_B}{1-\delta_A\delta_B}$

Sketch of a Proof

- Suppose A were to set $x < x_1^R$, holding all else constant
- B would accept, and A would be worse off than in equilibrium
- Suppose A were to set $x > x_1^R$
- B would reject and A would be worse off than in equilibrium
- $u_A(eqm) \geq u_A(x > x_1^R) \Leftrightarrow \frac{x_1^R}{1-\delta_A} \geq \delta_A \frac{x_2^R}{1-\delta_A}$
- Equivalent to $\frac{1-\delta_B}{1-\delta_A\delta_B} \geq \delta_A \frac{\delta_A(1-\delta_B)}{1-\delta_A\delta_B}$, which must be true
- Similar reasoning applies to B

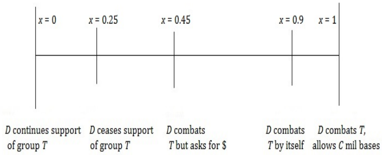
Implications

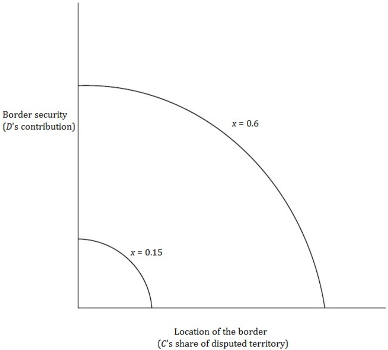
- Given assumptions of the model
  - Agreements reached immediately
  - The more patient a player is, the more they get
  - If both infinitely patient, even split in equilibrium
  - If equally but not infinitely patient, first-mover advantage
- Relaxing the assumptions
  - Being patient can be seen as having an attractive inside option
  - Less valuable than having an attractive outside option
  - Incomplete information can lead to delay

A Baseline Model of Crisis Bargaining

- C sets $x \in [0, 1]$
- D can either accept or reject
  - If D accepts, game ends peacefully
  - $u_C(\text{peace}) = x$ and $u_D(\text{peace}) = 1 - x$
  - If D rejects, game ends in war
  - $u_C(\text{war}) = w - c_C$ and $u_D(\text{war}) = 1 - w - c_D$

Divisibility





Analysis

- D accepts iff $u_D(\text{peace}) \geq u_D(\text{war})$
  - Equivalent to $x \leq \hat{x}$
  - Where $\hat{x} \equiv w + c_D$
- Does C make largest acceptable demand?
  - Iff $u_C(\text{peace}|x = \hat{x}) \geq u_C(\text{war})$
  - $\Rightarrow \hat{x} \geq w - c_C$
  - $\Rightarrow w + c_D \geq w - c_C$
  - $\Rightarrow c_C + c_D \geq 0$
  - This must be true

Bargaining with Distortions

- Partial outcomes more than proportionally decreased in value
  - $u_C(s) = s^\gamma$ and $u_D(s) = (1 - s)^\delta$
  - Where $\gamma > 1$ and $\delta > 1$ are C's and D's levels of distortion
  - Implies $u_C(s) < s$ and $u_D(s) < 1 - s$ for any $s \in (0, 1)$
  - But $u_C(s) = s$ and $u_D(s) = 1 - s$ if $s \in \{0, 1\}$
- Let w be C's probability of winning everything
  - Note that $u_C(\text{war})$ is still $w - c_C$
  - Proof: $w(1^\gamma - c_C) + (1 - w)(0^\gamma - c_C) = w - c_C$
  - But $u_C(\text{peace}) = x^\gamma$ is less than x unless $x = 1$
  - Similarly D's peace (but not war) payoff distorted

D's Acceptance Rule

- D accepts iff $u_D(\text{peace}) \geq u_D(\text{war})$
- $\Rightarrow (1-x)^\delta \geq 1 - w - c_D$
- We can again say D accepts iff $x \leq \hat{x}$
- Where $\hat{x} \equiv 1 - \sqrt[\delta]{1 - w - c_D}$

- For example, let $w = 0.5$ and $c_D = 0.2$

δ	$\hat{x}$
1	0.70
2	0.45
3	0.33
4	0.26

C's Choice of x

- C sets $x = \hat{x}$ iff $u_C(\text{peace}|x = \hat{x}) \geq u_C(\text{war})$
- $\Rightarrow \hat{x}^\gamma \geq w - c_C$
- $\Rightarrow (1 - \sqrt[\delta]{1 - w - c_D})^\gamma \geq w - c_C$
- This need not be true, and is less likely to hold as $\delta \uparrow$,$\gamma \uparrow$

- For example, let $w = 0.5$, $c_C = 0.2$, $c_D = 0.2$

		$\delta =$		
	1	2	3	4
$\gamma =$ 1	peace	peace	peace	war
2	peace	war	war	war
3	peace	war	war	war
4	war	war	war	war

Implications

- Suggests that war is more likely when
  - C, D divided by culture, religion, historic grievances
  - Leaders face reelection soon
- These possibilities have been examined statistically
  - Culturally dissimilar dyads experience fewer wars
  - History of conflict does not appear to promote conflict
  - Democrats less likely to go to war as elections approach
- Even bigger problem
  - No reason to expect γ, δ to fluctuate dramatically during war
  - If anything, they should increase
  - Yet majority of interstate wars end w/ negotiated agreement

Bargaining in the Face of Uncertainty

- Now assume C only knows $c_D \sim U(0, \overline{c}_D]$
- Must find x that maximizes
  $u_C(x) = \int_0^{x-w} w - c_C \; dc_D + \int_{x-w}^{\overline{c}_D} x \; dc_D$
- Or x that maximizes $(\frac{x-w}{\overline{c}_D})(w - c_C) + (\frac{\overline{c}_D - (x-w)}{\overline{c}_D})x$
- $\Rightarrow x^* = w + \frac{\overline{c}_D - c_C}{2}$
- D rejects with probability $\frac{\overline{c}_D - c_C}{2}$

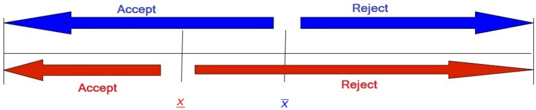
Uncertainty about Martial Effectiveness

- Everything the same as in basic model except
  - We now assume $w = \dfrac{e_C m_C}{e_C m_C + e_D m_D}$
  - And C does not know D's martial effectiveness
  - Only knows $pr(e_D = \underline{e}_D) = \phi$ and $pr(e_D = \overline{e}_D) = 1 - \phi$
  - Which implies $pr(w = \overline{w}) = \phi$ and $pr(w = \underline{w}) = 1 - \phi$

D's Acceptance Rule

- Still true that D accepts iff $u_D(\text{peace}) \geq u_D(\text{war})$
  - Blue type accepts iff $x \leq \overline{x}$
  - Red type accepts iff $x \leq \underline{x}$
  - Where $\overline{x} \equiv \overline{w} + c_D$ and $\underline{x} \equiv \underline{w} + c_D$

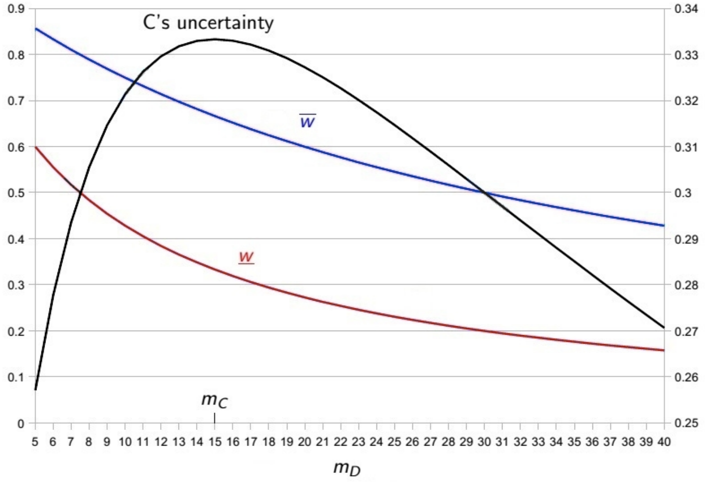
- C can readily infer the following
  - $pr(D \text{ accepts}) = 1$ if $x \leq \underline{x}$
  - $pr(D \text{ accepts}) = \phi$ if $\underline{x} < x \leq \overline{x}$
  - $pr(D \text{ accepts}) = 0$ if $x > \overline{x}$



C's Choice of x

- When C sets $x = \underline{x}$
  - Good news: probability of war is zero
  - Bad news: possible that D would have accepted $\overline{x}$
- When C sets $x = \overline{x}$
  - Good news: if D accepts, C gets best achievable outcome
  - Bad news: risks war
- When does C prefer $\underline{x}$ to $\overline{x}$?
  - $u_C(x = \underline{x}) = \underline{x}$
  - $E(u_C(x = \overline{x})) = \phi\overline{x} + (1 - \phi)(\underline{w} - c_C)$
  - $u_C(x = \underline{x}) \geq E(u_C(x = \overline{x}))$ holds iff $\phi \leq \hat{\phi}$
  - Where $\hat{\phi} \equiv \dfrac{c_C + c_D}{\overline{w} - \underline{w} + c_C + c_D}$

Parity and Uncertainty



Data

- Observations: all dyad-years from 1821 to 1913, 1946 to 2007
- y: outbreak of war w/ 2 states on opp sides
  - Taken from Correlates of War interstate war data
  - Excludes those who suffered $<$10% of fatalities on their side, unless that state fought alone for an extended period
- xs: Parity of Milcap, Total Cost
  - Parity of Milcap $= \frac{m_L}{m_L + m_H}$ where m_L is smaller m score
  - Total Cost based on energy consumption, distance

Results

	War
Parity	$+^*$
Total Cost	$-^*$