Game Theory

Phil Arena

Incomplete Information II

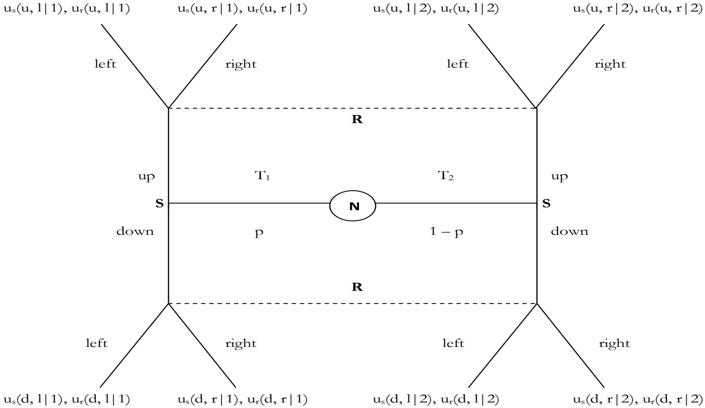
Introduction

- Two goals for this session
  1. Demonstrate identification of equilibria in signaling games
  2. Application: education

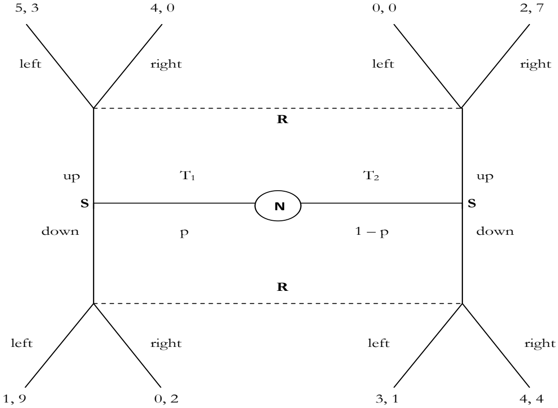
Signaling

- Model begins with Nature selecting a type of Sender
- Sender chooses one of two signals
- Receiver chooses one of two responses
- Receiver does not know which type was selected by Nature
- But may be able to infer S's type from her behavior
- Actions w/ no literal communication can still send signals

Generic Signaling Game



Example


5, 3
4, 0
0, 0
2, 7
left
right
left
right
R
up
T₁
T₂
up
S
N
S
down
p
1 − p
down
R
left
right
left
right
1, 9
0, 2
3, 1
4, 4

Identifying Equilibria

- Equilibria checked through proof by contradiction
- Three broad families of equilibria to check
- Pooling equilibria: both types of S send same signal, posterior beliefs same as prior beliefs
- Separating equilibria: each type of S sends a different signal, posterior beliefs 0 or 1, reflect true state
- Semi-separating: types sometimes, but not always, send same signals, posterior beliefs between priors and full revelation
- Must go through each to determine if there exist conditions under which it can be a PBE
- Generally expressed as $(s_{R|u}, s_{R|d}; s_{S|T_1}, s_{S|T_2}; p_u, p_d)$

Pooling on Up

- If both types play up, R's posterior belief that S is T_1 given that they played up, which we can call p_u, is simply p
- Thus, $E(u_R(l|u)) = p(3) + (1-p)(0) = 3p$, $E(u_R(r|u)) = 7 - 7p$, and R plays left iff $p \geq 0.7$
- Can S profit by deviating and playing down?
- We cannot know what R's posterior belief after observing down is, since Bayes' rule is undefined
- But R must hold a belief, so see if $\exists\ p_d \ni\ S$ won't deviate
- When $p \geq 0.7$, T_2 prefers down $\forall\ p_d$, since $E(u_S(u, l|2)) = 0$
- When $p < 0.7$, T_2 again prefers down, so there cannot be any PBE where S pools on up

Pooling on Down

- When S pools on down, $p_d = p$, p_u undefined
- $E(u_R(l|d)) = p(9) + (1-p)(1) = 8p + 1$,
  $E(u_R(r|d)) = 4 - 2p$, and R plays left iff $p \geq 0.3$
- Again, R's posterior belief cannot be determined by Bayes' rule, so any belief is weakly consistent
- If $p \geq 0.3$, T_1 prefers up to down $\forall\ p_u$, since $E(u_S(d, l|1)) = 1$ and worst payoff for playing up is 4
- If $p < 0.3$, T_1 again prefers up, because $E(u_s(d, r|1)) = 0$
- There are no values of p_u for which S prefers to stick to the equilibrium strategies
- There can be no PBE where S pools on down

Separating, T_1 plays Up

- If T_1 plays up and T_2 down, $p_u = 1$ and $p_d = 0$
- R prefers left to right after observing up, since $3 > 0$
- R prefers right to left after observing down, since $4 > 1$
- Given that R holds the beliefs she holds and will play the strategies that follow from those beliefs, does S want to abide by the strategies we've supposed?
- T_1 cannot benefit from switching to down, as she is already receiving her best payoff
- T_2 likewise cannot improve her payoff, as she too is already doing the best she can
- Therefore, $(L, R; U, D; 1; 0)$ constitutes a PBE

Separating, T_1 plays Down

- If T_1 plays down and T_2 up, $p_u = 0$ and $p_d = 1$
- R prefers right to left after observing up, since $7 > 0$
- R prefers left to right after observing down, since $9 > 2$
- T_1 prefers to switch to up, receiving a 4 instead of 1
- T_2 prefers to switch to down, receiving 3 instead of 2
- But this creates a contradiction, as $p_u = 0$, $p_d = 1$ would no longer be consistent with Bayes' rule
- There cannot a separating PBE where T_1 plays down, T_2 up

Setup

- Individual i decides whether to obtain a degree at cost κe
- After observing i's choice, F hires i into either position 1 or 2
- If i hired into position 1, i and F both receive 1
- If i hired into position 2, i decides whether to invest in acquiring new skills at cost $\kappa\iota$
- If so, i paid salary $\overline{s}$
- If i hired into position 2 but does not invest, paid $\underline{s}$
- If i hired into position 2, F receives β iff i acquires skills, 0 otherwise
- Where $1 < \underline{s} < \overline{s}$, $\beta > 1$
- i knows value of κ, F does not
- But F knows $\kappa \sim U(0, 1]$

Semi-separating Equilibrium

- F hires i into position 2 iff i obtains degree
- i obtains degree iff $\kappa \leq \overline{\kappa}$ and acquires new skills iff $\kappa \leq \underline{\kappa}$
- F's strategy optimal iff $(\underline{\kappa}/\overline{\kappa})\beta \geq 1$
- i's optimal when
    - $\overline{\kappa} = \dfrac{\underline{s} - 1}{e}$
    - $\underline{\kappa} = \dfrac{\overline{s} - \underline{s}}{\iota}$
    - Requires $\underline{\kappa} < \overline{\kappa}$

Implications

- Comparative statics
  - As $e \downarrow$, more types obtain degree
  - But less and less likely that F pays premium
- Empirical evidence
  - Sheepskin effect
  - Ivy league grads hired by Wall Street irrespective of major