Game Theory

Phil Arena

Incomplete Information I

Introduction

- Two goals for this session
  1. Application: trade
  2. Application: teenage angst

Bayesian Equilibria

- Equivalence between imperfect and incomplete information
- When behavior depends on beliefs, NE/SPNE no longer suffice

Bayesian Nash Equilibria

Each type of each player simultaneously adopts their best response to the other player's best response, given their beliefs about the relative probabilities of facing each type of the other player.

Perfect Bayesian Equilibria

Each type of each player adopts sequentially rational strategies given their beliefs, and beliefs are weakly consistent with Bayes' Theorem (i.e., updated where possible).

A Model of Trade, Trust, and Exploitation

	allow	block
allow	β, β	βe_1, τ_2
block	τ_1, βe_2	0, 0

- Player i knows
  - The value of β, e_i, e_j, and τ_i
  - That $\beta > 0$, $e_i, e_j \in [-1, 1]$
  - $pr(\tau_j = \underline{\tau}_j) = \phi_j$
  - $pr(\tau_j = \overline{\tau}_j) = 1 - \phi_j$

Analysis

- Many equilibria possible
- But not all of them interesting
- Only in some equilibria do types behave differently
- When it does not, players' uncertainty is irrelevant
- We'll focus on the only BNE where play depends on type
- In it, both players allow iff they are the blue type
- Which immediately implies $\underline{\tau} < \beta < \overline{\tau}$

Proof

- For blue type
  - Allow iff $\phi_j(\beta) + (1 - \phi_j)(\beta e_i) \geq \phi_j(\underline{\tau_i}) + (1 - \phi_j)(0)$
  - Must be true if $e_i \geq 0$
  - If $e_i < 0$, true iff $\phi_j \geq \hat{\phi}_j$
  - Where $\hat{\phi}_j \equiv \dfrac{\beta e_i}{\underline{\tau_i} - \beta + \beta e_i}$
- For red type
  - Allow iff $\phi_j(\beta) + (1 - \phi_j)(\beta e_i) \geq \phi_j(\overline{\tau_i}) + (1 - \phi_j)(0)$
  - Cannot be true if $e_i < 0$
  - If $e_i \geq 0$, true iff $\phi_j > \hat{\phi}_j$
  - Where $\hat{\phi}_j \equiv \dfrac{\beta e_i}{\overline{\tau_i} - \beta + \beta e_i}$

Implications

- Trust is only sometimes necessary for mutual cooperation
- When imbalanced trade tolerable, trust unnecessary
- As the benefits of cooperation ↑, trust less important

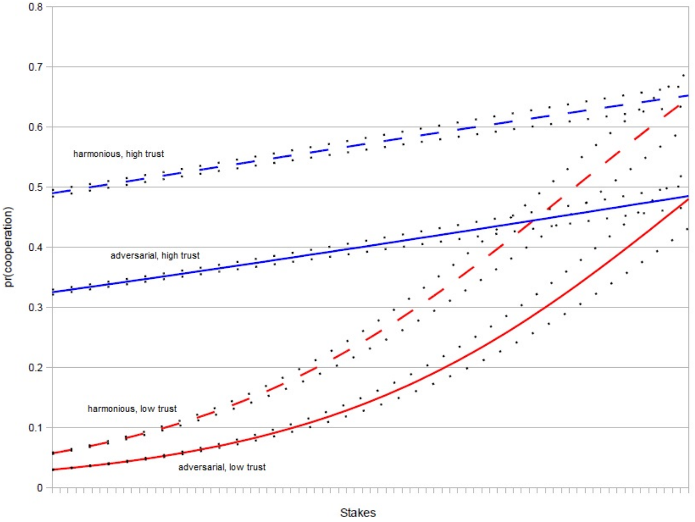
- Proof: $\frac{\partial \hat{\phi}_j}{\partial \beta} = \frac{(\underline{\tau}_i - \beta + \beta e_i)(e_i) - (\beta e_i)(-1 + e_i)}{(\underline{\tau}_i - \beta + \beta e_i)^2}$
- $\Rightarrow \frac{e_i \underline{\tau}_i}{(\underline{\tau}_i - \beta + \beta e_i)^2}$

- Given $e_i < 0$, $\tau < \beta$, this must be negative

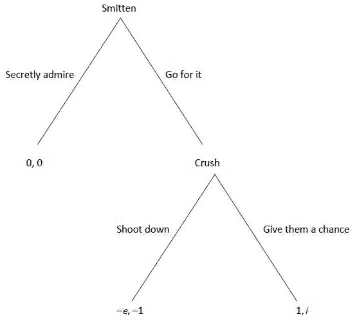
Empirical Evaluation

- Observations: all dyad-years from 1870 to 1913, 1950 to 2005
- Dependent variable: cooperative trade relations
  - Equals 1 iff 3 conditions met ($\approx$ 35% of cases)
  - 1: Imports from 1 to 2 $>$ 10 mil current US \$
  - 2: Imports from 2 to 1 $>$ 10 mil current US \$
  - 3: Larger flow $<$ 75% of bilateral trade volume
- Independent variables: trust (ϕ), harmonious (e), stakes (β)
  - Trust equals 1 iff 1 has embassy in 2 and 2 in 1
  - Harmonious equals $\frac{energyconsump_L}{energyeconsump_L + energyconsump_H}$
  - Stakes equals $\frac{population_1 \times population_2}{ln(distance)}$

Results


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
pr(cooperation)
harmonious, high trust
adversarial, high trust
harmonious, low trust
adversarial, low trust
Stakes

Teenage Angst Revisited



- Assume Crush knows value of i, but Smitten does not
- Smitten only knows $pr(i = \bar{i}) = \phi$ and $pr(i = \underline{i}) = 1 - \phi$

Analysis

- Suppose $\underline{i} < -1$ and $\bar{i} > 0$
- Then $u_S(admire) \geq E(u_S(go)) \Leftrightarrow 0 \geq (1-\phi)(-e) + \phi(1)$
- Holds iff $\phi \leq \hat{\phi}$, where $\hat{\phi} \equiv \frac{e}{1+e}$
- Smitten more likely to go for it when optimistic
- The more sensitive their ego, the more optimistic they must be
- Proof: $\frac{\partial \hat{\phi}}{\partial e} = \frac{1}{(1+e)^2}$, which is positive
- $\lim_{e \to \infty} \hat{\phi} = 1$, $\lim_{e \to 0} \hat{\phi} = 0$