Game Theory

Phil Arena

Repeated Games

Introduction

- Three goals for this session
    1. Discuss finite games
    2. Contrast with infinitely-repeated games
    3. Application: inter-ethnic cooperation

Finite Games

- Can Grim Trigger induce cooperation in PD played T times?
  - GT is a strategy whereby i:
  - cooperates in the first round;
  - cooperates in subsequent rounds iff j never defected;
  - defect otherwise

- No, it cannot
  - In final period, neither player willing to cooperate
  - That means there can be no punishment for defecting at $T - 1$
  - And that means there is no punishment at $T - 2$
  - This logic extends all the way back to the first period

Credible Threats Induce Cooperation

	L	M	R
T	8, 8	0, 0	1, 9
M	0, 0	5, 5	0, 0
B	9, 1	0, 0	3, 3

- Suppose this game is played twice
- Let $\sigma_1 = T$ in period 1, M in 2 iff L in 1, B otherwise
- Let $\sigma_2 = L$ in period 1, M in 2 iff T in 1, R otherwise
- This combination of strategies comprises a SPNE
- $u_1(\sigma_1|\sigma_2) \geq u_1(B, B|\sigma_2) \Leftrightarrow 8 + 5 \geq 9 + 3$
- $u_2(\sigma_2|\sigma_1) \geq u_2(R, R|\sigma_1) \Leftrightarrow 8 + 5 \geq 9 + 3$

Time Preferences

- Constant discounting
    - Utilities weighted by $\delta^t \in (0, 1)$
    - Where $t \in [0, \infty)$ is # of periods in the future
    - Ex: $x + \delta x + \delta^2 x$ if x received in each of three periods
    - Infinite stream of x is worth $\dfrac{x}{1 - \delta}$

- Recursion
    - Let $V_i(s_t, h_t)$ be i's continuation value given choice of strategy s_t at time t and history of play h_t
    - Let $u_i(o_t)$ be i's immediate utility for outcome resulting from s_t
    - We can write $V_i(s_t, h_t) = u_i(o_t) + \delta V_i(s_{t+1}, h_{t+1})$

Hyperbolic Discounting

- Utilities weighted by $\dfrac{1}{(1 + \alpha t)^{\frac{\gamma}{\alpha}}}$, where $\gamma > 0$, $\alpha > 0$
- Ex: $x + \dfrac{x}{(1+\alpha)^{\frac{\gamma}{\alpha}}} + \dfrac{x}{(1+2\alpha)^{\frac{\gamma}{\alpha}}}$
- Explains time consistency problems
- Common in behavioral economics

Infinitely-Repeated Generic Cooperation Game

	c	d
c	3, 3	1, $2 + \tau_2$
d	$2 + \tau_1$, 1	2, 2

- Suppose $\tau_1 > 1$ and $\tau_2 > 1$
- (c,c) not possible in one-shot play
- But what if the game is infinitely repeated?

Analysis

- Now, Grim Trigger can induce cooperation
  - $u_i(\text{GT}_i|\text{GT}_j) \geq u_i(\text{d}|\text{GT}_j)$ is equivalent to:

  - $3 + \delta 3 + \delta^2 3 + \ldots \geq 2 + \tau_i + \delta 2 + \delta^2 2 + \ldots$

  - $\Rightarrow \dfrac{3}{1 - \delta} \geq 2 + \tau_i + \delta(\dfrac{2}{1 - \delta})$

  - Simplifies to $\tau_i \leq \dfrac{1}{1 - \delta}$

Folk Theorem

In every infinitely-repeated game with discounted payoffs, there exists some δ that can sustain any equilibrium in which the players adopt strategies that deliver payoffs that exceed their minimax payoff. Such equilibria are sustained by mutual threats to punish deviations therefrom by reverting to the strategy that imposes the other player's minimax payoff.

Application: Inter-ethnic Cooperation

- Two ethnic groups, A and B
- Each period, play GCG with randomly selected partner
- Partner might be from either ethnic group
- Everyone knows history of play for every member of own group
- No one knows history for individual members of other group
- Both groups know when they've been defected against

Informal Analysis

- Spiral equilibrium
  - Any inter-ethnic defection brings punishment for entire group
  - Individual-level punishments deter within-group defection
  - Cooperation w/in and b/w groups possible in equilibrium

- In-group policing equilibrium
  - Inter-ethnic defection goes unpunished by out-group
  - But is punished by other members of the same group
  - Within-group defection also punished by group
  - Interactions b/w groups cannot be too frequent

Implications

- Interactions b/w groups cannot be frequent in either eqm
- Nor can they be too rare in the first equilibrium
- Assumes behavior strictly self-interested
- Ignores impact of interactions on preferences
- Also assumes interactions cannot be forced
- Yet there is ethnographic evidence of the mechanisms